APPLICATION OF FIRST ORDER DIFFERENTIAL EQUATION IN TEMPARATURE PROBLEMS

A Group Project Report

Submitted in Partial fulfillment of the requirements for the award of Degree of

BACHELOR OF SCIENCE IN MATHEMATICS

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CERTIFICATE

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This is to certify that the project reported entitled **“APPLICATION OF FIRST ORDER DIFFERENTIAL EQUATION IN TEMPARATURE PROBLEMS”** Submitted to **Sri G.V.G. Visalakshi College for Women(Autonomous), Udumalpet,** Affiliated to Bharathiyar University, in partial fulfillment of the requirements for the award Degree of **BACHELOR OF SCIENCE IN MATHEMATICS”** is a record of original project work done by A.P.Aashika,T.Anusuyadevi,M.Kokila,R.Madhumitha,R.Nandhini,K.Reshma during the period August 2020 - December 2020 of their study in the Department of Mathematics, Sri G.V.G. Visalakshi College for Women (Autonomous), Udumalpet under my supervision and guidance and the project has not formed on the basis for award of any degree/diploma/associate ship/ fellowship or other similar title to any other candidate of any University.

Submitted for viva –voice held on ________________

Sri G.V.G. Visalakshi College For Women,Udumalpet.

Head of the Department **Signature of the guide**

Internal Examiner **External Examiner**

ACKNOWLEDGEMENT

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First foremost, we record our sincere gratitude to the almighty for her blessings for the successful completion of the project

We take immense pleasure in expressing our thanks to the management for their continuous support throughout the project. We are extremely grateful to the principal **(i/c) Mrs. S. Kalaisevi M.sc., B.Ed., M.Phill., PGDCA.,** SRI GVG Visalakshi college for women (Autonomous)for providing all facilities to do this project.

We take privilege in expressing our sincere, heartfelt thanks and gratitude to **Mrs.B. Pushpa M.sc., MPhill, DCA.,** Head of the department , Department of Mathematics (S.F) for her continuous support throughout the project.

We record our profound sense of indebtness to our guide **Mrs.B. Pushpa M.sc., MPhill., DCA.,** for taking a keen interest and given us valuable suggestions, guidance in all aspects of this group project. We express our deep gratitude and humble thanks to the faculty members of the department of their valuable suggestions and lending helping hand for the successful completion of our project.

Finally we would like to thank our parents and their friends who have contributed much to complete the group project work on time

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DECLARATION

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We hereby declare that the project entitled **"APPLICATION OF FIRST ORDER DIFFERENTIAL EQUATIONS IN TEMPERATURE PROBLEM"** submitted to **Sri G.V.G Visalakshi college for Women (Autonomous),Udumalpet,** Affiliated to Bharathiyar University, In Partial fulfillment of the requirements for the award of Degree of **"BACHELOR OF SCIENCE IN MATHEMATICS"** is a record of original project work done by us during the period June 2020 – October 2020 of our study under the supervision and guidance of **Mrs. B.PUSHPA M.Sc., M.Phil., DCA.,** and the project has not formed on the basis for the award of any degree/diploma/associate ship/fellowship or other similar title to any other candidate of any university.

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CONTENTS

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ABSTRACT

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In this project the application of first order differential equation in temperature are studied. The method of separation of variables, Newton's law of cooling are used to find the solution of the temperature problems that requires the use of first order differential equation and these solution are very useful in mathematics, biology, and physics especially in analyzing problems involving temperature which requires the use of Newton's law of cooling.

INTRODUCTION

CHAPTER I

INTRODUCTION

A first order differential equation is an equation that contain only first derivative, and it has many application in mathematics, physics, and engineering many other subjects. A first order differential equation plays a vital role in physics that includes a temperature problem which requires the use of Newton's law of cooling of a particular substance.
According to some historians of mathematics the study of differential equation began in 1675, when Gottfried Wilhelm Von Leibniz wrote the Equation

$$\int x dx \;=\; \frac{1}{2}x^2 \tag{1}$$

Isaac Newton classified first order differential equation into three classes.

$$\begin{aligned} &\frac{dy}{dx} = f(x) \\ &\frac{dy}{dx} = f(x, y) \\ &x\frac{du}{dx} + y\frac{du}{dx} = u \end{aligned} \tag{2}$$

The first two classes contain only ordinary derivatives of one or more dependent variables with respect to single independent variable and are known today as ordinary differential equations

Michael et al., stated that "the range of water temperature found on the earth encompasses both the freezing point (0oC for fresh water and -1.9 0oC for sea water) and boiling point (100 0oC fresh water, 102 0oC for sea water).

Much the earth's free water is contained in the oceans at the temperatures toward the low end of this range, remaining nearly frozen at an average temperature of approximately 1 0oC. very warm waters are found at a few locations on the point. a small amount of the earth's water, around 0.5% is contained in ground water with typical temperature similar to local average annual air temperatures. Lakes and rivers containing only around 0.01% of the earth free water have temperature ranges between 0 and 40 0oC, in which most biological activity occur, shallow lakes and rivers in one climate can reach temperature of 40 0oC, however, maximum temperature of most lakes and rivers in one climate can reach temperature of 40 0oC , however, maximum temperatures of most lakes and stream are somewhat less than this extreme".

Longini et al, studied the effectiveness of using targeted antiviral prophylaxis to contain an epidemic of the flu before an effective vaccine is found.

Using mathematical model they predicted that without any form of intervention an influenza illness attack at a growth rate of 33% of the population and an influence, death rate of 0.58 per one thousand persons will occur but with targeted antiviral prophylaxis if 80% of the exposed population is maintained on prophylaxis for up to eight week pandemic will be continued the virus.

The aim and objective of this project is to use first order differential equation in solving some problems that are in temperature problems. The scope of this project is to give an insight in to the application of first order differential equations in temperature problems.

This project is limited to the first order differential equation only.

DIFFERENTIAL EQUATIONS

CHAPTER II
DIFFERENTIAL EQUATIONS

A Differential equation is an equation that involves one or more derivatives of differentials that is any equation containing differential coefficients is called Differential equation. It is also defined as an equation containing the derivatives or Differentials of one or more dependent variables, with respect to independent variable

$$\frac{dy}{dx} + 4xy = \cos x \tag{3}$$

$$\left(\frac{dy}{dx} = \text{derivative}\right)$$

CLASSIFICATION OF DIFFERENTIAL EQUATION

Differential equation is classified according to three properties which include;

i. Classification by type

ii. Classification by order and degree

iii. Classification as linear or non linear differential equation

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i. Classification by type

If an equation contains only ordinary derivatives of one or more dependent variables, with respect to a single independent variable is then called Ordinary differential equation

$$f\,(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \ldots\ldots) = 0 \qquad (4)$$

Defines an ordinary differential equation for y (the dependent variable) in terms of x (the independent variable).

For example

$$\frac{dy}{dx} - 3y = 0 \qquad (5)$$

ii. Classification by order and degree

The order of differential equation is the highest differential coefficients contained in it.

$$\frac{dy}{dx} - 3y = 0 \qquad (6)$$

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = \sin x \qquad (7)$$

are the examples of first and second order ordinary differential equations .
The degree of any differential equation is the exponent of the highest powers

For example;

$$\left(\frac{dy}{dx}\right)^4 + y^3 + \left(\frac{dy}{dx}\right)^3 = 2 \qquad (8)$$

is of degree 4

iii. Classification as linear or nonlinear differential equations

Generally a linear differential equation has the form:

$$a_n(x)\frac{d^n x}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x) \tag{9}$$

It should be noted that linear differential equations are characterized by two properties:

(i) The dependent variable y and all its derivatives are of the first degree that is the power of each term involving y is 1.
(ii) Each coefficient depends only on the dependent variable x.

Differential equation is said to be non linear if the equation (9) together with the two given properties are not satisfied.

Examples of linear differential equations are equations of the form:

$$\frac{dy}{dx} - 3y = 0 \tag{10}$$

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = e^x$$

Examples of nonlinear differential equations are equation of the forms:

$$\frac{dy}{dx} - xy^{\frac{1}{2}} = 0 \tag{11}$$

are non linear ordinary differential equations.

SOLUTION OF DIFFERENTIAL EQUATION

CHAPTER III

SOLUTION OF DIFFERENTIAL EQUATION

The first order differential equation has a variety of methods in finding the solution of equation which include.

- Variable separable
- Equation reducible to variable separable
- Homogeneous equations
- First order exact differential equations

Variable separable

A first order differential equation can be solved by integration if it's possible to collect all y terms with dy and all x terms with dx. That is if it's possible write the equation in the form.

$$f(y)dy + g(x)dx = 0 \qquad (12)$$

Then the general solution is

$$\int f(y)\, dy + \int g(x)dx = c \qquad (13)$$

Where C is an arbitrary constant.

Example 1:

Solve the differential equation $\frac{dy}{dx} = \frac{x^2+2}{y}$ (14)

Solution:

Arrange the equation in the form

$f(y)\, dy + g(x)\, dx = 0$ (15)

Multiply both sides by *y*

$y \frac{dy}{dx} = x^2+2$

$y\, dy = (x^2+2)\, dx$ (16)

$ydy-(x^2+2)dx=0$

Now we've

$f(y)=y$, $g(x) = -(x^2+2)$ (17)

Therefore

$\int y\,dy + \int (-(x^2+2))\,dx = c$

$\int y\,dy - \int (x^2+2)dx = c$

$\frac{y^2}{2} - [\frac{x^3}{3} + 2x] = c$

$\frac{y^2}{2} = c + \frac{x^3}{3} + 2x$

$y^2 = 2c + 2\frac{x^3}{3} + 4x$

Let $2c = k$

$y^2 = 2\frac{x^3}{3} + 4x + k$

$y = \pm\sqrt{2\frac{x^3}{3} + 4x + k}$

Solving for y explicitly we obtain the two solutions as

$y = \sqrt{\frac{2}{3}x^3 + 4x + k}$ and (18)

$y = -\sqrt{\frac{2}{3}x^3 + 4x + k}$

Equation reducible to be variable separable:

Differential equation of the first order cannot be solved directly by the variable separable method. But by some substitution, we can reduce it to a differential equation with separable variable.

Homogeneous equations:

A first order differential equation $\frac{dy}{dx} = f(x,y)$ is called homogeneous equation, if the right side satisfies the condition.

$f(tx,ty) = f(x,y)$ for t

First order exact differential equation:

A first order differential equation is an equation of the form $F(t,y,y)=0$……. The term “first order” means that the first derivative of y appears, but no higher order derivatives do.

APPLICATION OF FIRST ORDER DIFFERENTIAL EQUATION IN TEMPERATURE PROBLEM

CHAPTER IV

APPLICATION OF FIRST ORDER DIFFERENTIAL EQUATION IN TEMPERATURE PROBLEM

The few common application of some elementary differential equation in temperature are discussed

Temperature problems in differential equation

Assuming that the question involves the temperature (T) of a certain body placed in a medium of constant temperature (M) and as time (t) varies so does T. In this case Newton's law of cooling tells us the following.

$$\frac{dT}{dt} = -k(T - M) \qquad (19)$$

"The rate of change of temperature of the body is proportional to the difference in temperature of the body and the medium in which it is placed".

The another one is, greater the difference in temperature between the system and surroundings, more rapidly the **heat is transferred** i.e, the more rapidly the body temperature of body changes. Newton's law of cooling formula is expressed by $\mathbf{T(t) = T_s + (T_0 - T_S)\, e^{-kt}}$

Example 1

A metal bar at a temperature of 100^0F is placed in a room at constant temperature of 0^0F, if after 20 minutes the temperature of the bar is 50^0F, find

(a)The time it will take the bar to reach the temperature of 25^0F, and

(b)The temperature of the bar after 10 minutes.

Solution

$T=100^0$ F,

$M=0^0$ F

Using equation 19

$$\frac{dT}{dt} = -k(T - M)$$

$$\frac{dT}{dt} = -k(T - 0)$$

$$dT = -kT\, dt$$

$$\int \frac{dT}{T} = -\int k\,dt$$

$\ln(T) = -kt + c$

$T = e^{-kt+c}$

Now,

$T = ce^{-kt}$ (20)

Since $T=100^0F$, at $t=0$,(the temperature of the bar is initially 100^0F) and it follows from equation 20 above

$100 = c\, e^{-k(o)}$

This implies that

C=100

And so equation 20 becomes

$T=100e^{-kt}$ (21)

And at t=20, we are given that $T=50^0F$, hence from equation (21) ,

$50 = 100e^{-k(20)}$

$\frac{50}{100} = e^{-20k}$

$\ln(0.5) = e^{-20k}$

$k = \frac{ln(0.5)}{-20}$

k = 0.035

Now substituting the value of k into equation 21,

$T=100e^{-0.035t}$ (22)

Equation 22 represent the temperature of the bar at any time,

(a) To find t when $T=25^0F$, now from equation 22

$25=100e^{-0.035t}$

$\frac{1}{4} = e^{-0.035t}$

$\ln(0.25)=-0.035t$

Temperature,t = 39.6min.

Hence the 39.6 min will take the bar to reach the temperature of 25 F.

(b) Find T when t=10min, In equation 22

$t = 100e^{-0.035t}$

$t = 100e^{-0.035(10)}$

Temperature,t = 70.5 F.

Hence the 70.5 F of the bar after 10 min

T = 100* exp (-0.035*t)

Metal bar

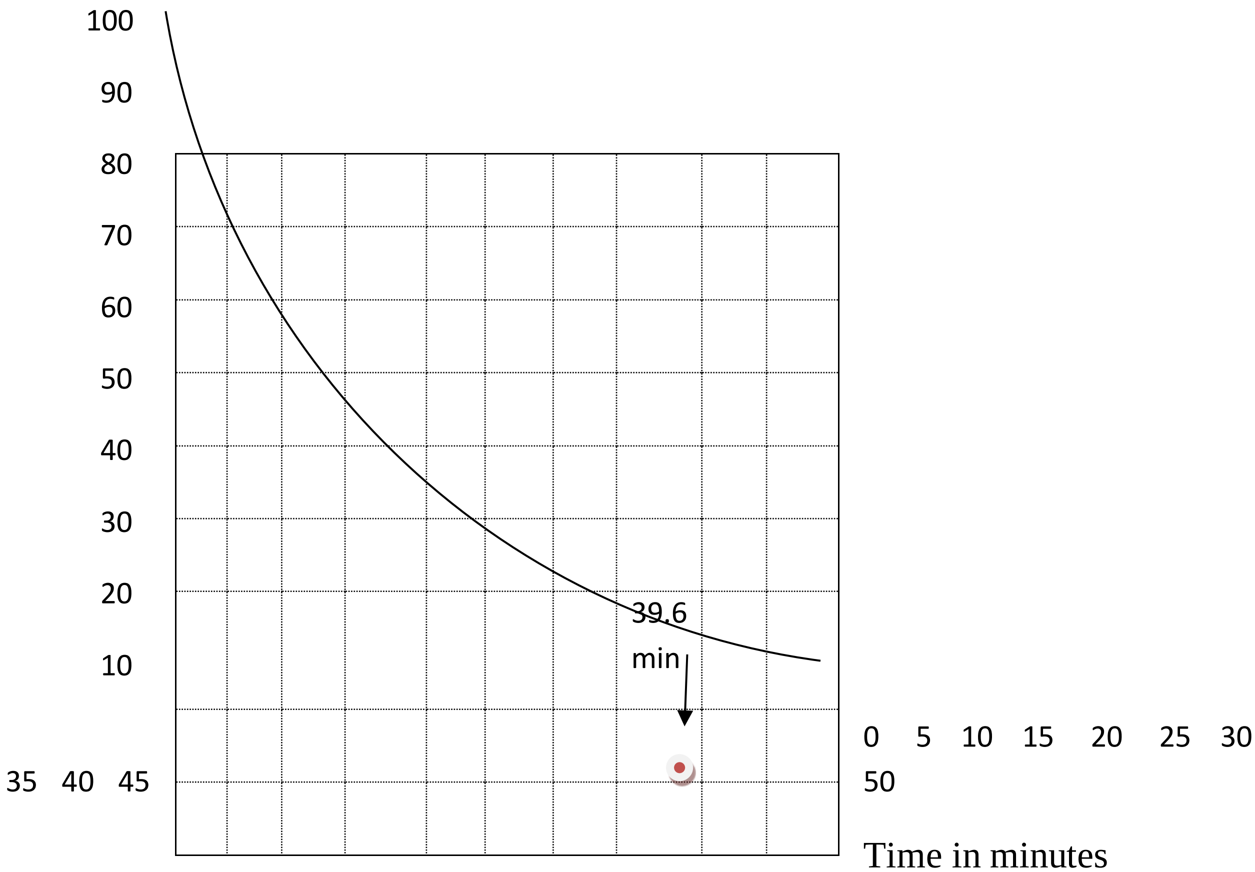

Figure 1: The above graph represents a graphical solution of the problem

The above graph explains the change of temperature of a metal bar in a room, at time (t), it also shows that is takes the temperature of the metal bar approximately 39.6 minutes to drops down to 25^0F, while the ambient temperature remains steady at 0^0F.

Example 2:

A boiling (100℃) solution is set on a table where the room temperature is assumed to be constant at 20℃ , the solution cooled to 60℃ after five minutes

(a) Find a formula for the temperature (T) of the solution, t minutes after it is placed on the table.
(b) Determine how long it will takes for the solution to cool to 22℃.

Solution:

(a) To find an explicit formula for T in terms of t, but this kind of problem requires the use of Newton's law of cooling which states that

$\frac{dT}{dt}$ = k (T − M) for some constant k

Where M is the constant temperature.

Solving the equations using variable separable

dT = k (T − 20) dt For some constant k

dT= k (T −20) dt

$\frac{dT}{T-20}$ =kt+c

Integrate both sides of the equation

dT = k (T−20) dt

$\int \frac{dT}{T-20} = \int kt + c$

in(T−20) = kt+c

$\text{T}-20 = e^{kt+c}$

T = Ae^{kt}+20

where A = e^{c}

Since the initial temperature of the solution was 100℃ , T = 100℃

From T = Ae^{kt}+20

Substituting

T=100 and t=0,

100 =A$e^{k(0)}$+20

A = 80

So,

T = 80e^{kt}+20

Now using t = 5 minutes and T =60℃, to find k

60 = 80$e^{k(5)}$+20

40 = 80e^{5k}

$\frac{40}{80} = e^{5k}$

$\frac{1}{2} = e^{5k}$

Taken ln of both sides

5k = ln ($\frac{1}{2}$)

k = $\frac{1}{5}$ ln($\frac{1}{2}$)

k = $-$0.139

T = 80$e^{(-0.139)t}$+20(23)

(b) To find out what is t when T is 22℃,

T = 80$e^{(-0.139)t}$ +20

At T = 22℃ we have

22 = 80$e^{-0.139t}$ +20

Collecting like terms together,

22-20 = 80$e^{-0.139t}$

2 = 80$e^{-0.139t}$

$\frac{2}{80} = e^{-0.139t}$

$\frac{1}{40} = e^{-0.139t}$

Taken ln of both sides

$\ln(\frac{1}{40}) = -0.139t$

t = 27 minutes

Therefore after 27 minutes the temperature will reduce to 22℃ .

Example 3

A cup of coffee at 190^0F is left in a room of 70^0F. At time t=0, the coffee is cooling at 15^0F as per minute.

a)Find the function that models the cooling of the coffee.

b)How long will it take for the temperature to reach 143^0F?

solution,

a)

$T=190^0F$

$C=70^0F$

$T=0$

$T(t)=ae^{kt}+C$

$\frac{dT}{dt}$=-k(T-C)

$\frac{dT}{dt}=-15^0\frac{F}{min}$

Sub,

-15=-K(190-70)

-15=-K(120)

$K=\frac{15}{120}$

$K=\frac{1}{8}$

K= 0.125

$\frac{dT}{dt}= - 0.125(T-70)$

$\frac{dT}{T-70}= - 0.125dt$

By Integrating,

$\int\frac{dT}{T-70} = \int -0.125dt$

$\log(T-70) = -0.125t+C_1$

Taking a power,

$e^{\log(T-70)} = e^{(-0.125t+C_1)}$

$T - 70 = e^{-0.125t}\, e^{c_1}$

Let, $(e^{C1}=a)$

Another Constant

$T - 70 = ae^{-0.125t}$

$T = ae^{-0.125t}+70$

$(T(t) = ae^{Kt}+C$

$T(0) = ae^{-0.125t}+70$

$190 = ae^{-0.125t}+70$

$190 - 70 = ae^{-0.125t}$

$120 = ae^{-0.125t}$

$120 = ae^{-0.125(0)}$

$120 = ae^{0}$

$A = 120$

$T(t) = 120e^{-0.125t}+70$

b)

$T(t) = 143^0F$

$143 = 120e^{-0.125t}+70$

$143 - 70 = e^{-0.125t}$

$73 = 120e^{-0.125t}$

$\frac{73}{120} = e^{-0.125t}$

$\log(\frac{73}{120}) = \log e^{-0.125t}$

-0.125 t = log($\frac{73}{120}$)

T = $\frac{\log\ (\frac{73}{120})}{-0.125}$

t=3.98 minutes

The temperature reach is 3.98 minutes.

EXAMPLE 4

To find Body temperature using Newton’s cooling law:

Newton’s law of cooling T(t) = T_s+ $(T_0 - T_s)e^{-kt}$

Body temperature at 6 pm in 80℃ and at 7 pm it is 75℃ and the room temperature is 70℃ . To find the time of death.

SOLUTION:

T(t) = T_s+ $(T_0 - T_s)e^{-kt}$

75 = 70 + (80-70)$e^{-k(60)}$

5 = 10$e^{-k(60)}$

$\frac{1}{2}$ = $e^{-k(60)}$

log $\frac{1}{2}$ = log$e^{-k(60)}$

log$\frac{1}{2}$ = -k(60)

k = 0.0116

80 = 70 + (98.6 – 70)$e^{-0.0116t}$

10 = 28.6$e^{-0.0116t}$

log ($\frac{10}{28.6}$) = -0.0116t

-1.6508 = -0.0116t

t = $\frac{-1.6508}{-0.0116}$

t = 90.59 minutes (91 minutes)

Therefore time of death is 4.29 pm

CONCLUSION

CONCLUTION

In this project the application of first order differential equation in temperature problems were found. It is useful in mathematics and physics for instance in analyzing problems involving temperature problems which requires the use of Newton’s law of cooling. When dealing with temperature problem it is recommended to use Newton’s law of cooling and the most appropriate method in solving Newton’s law of cooling is variable separable.

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